Research Article

Yagya Kunds of one-hast (24 angul) with different shapes have equal volume

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Abstract. Yagya kund construction is the outcome of great research of ancient India. Indian scripture has given very sophisticated Vedic mathematical formulations for construction of Yagya kund. There are different types of shapes described for Yagya kund; Circular & Lotus, Semi-circular, Vulvar, Trigonal, Square, Pentagonal, Hexagonal, Heptagonal, Octagonal. Irrespective of shapes,all these Yagya kunds have same surface area. Based on the fact given in the literature, 1000 offerings (ahutis) require construction of BhuHastatmakaKund (1 hand or 24 angul long). In addition, height of the all one-hand long kunds are same. Hence, the present research tests the hypothesis that the volume should be same for all different shaped kunds. In the present study, the volume of 1 hast Yagya kund (24 angul) for all these shapes is calculated using the dimensions given in the scripture using available simple available mathematical formulas. Volume of all these kunds is compared with circular shape kund. The difference in the volume of different shapes is foundbelow 0.3% in all the kunds except for vulvar, pentagonal and octagonal shapes which is observed to be 7.48%, 1.76% and 2.83% respectively. The difference isdueto inappropriate mathematical formula for these complex structures having different angles in the slants and multiple sides of the bases.

Keywords. Yagya Kund, Shapes of Yagya Kund, Volume, Yagya Kund Construction, Angul

Introduction

Yagya kund construction is the outcome of great research of ancient India. Indian Sages (Rishis) studied and produced science of Yagya kund construction for various purposes benefiting all the living creatures, environment and the planet at large. To produce desirable outcomes Yagya kunds of different size and shapes were used (1). As per the book Shardatilak (2), the size of Yagya Kund should be considered on the basis of number of offerings to the Yagya kund irrespective of the shape of Yagya kund (3). For example, for the 1000 offerings (ahuties) BhuHastatmakaKund (1 hand long) is enough, for 10000 offerings DueHastatmaka(2 hand long) should be considered, and for 100000 offerings person should make ChaturHastKund (4 hand long), and if evenmore number of offerings are involved in Yagya the AashtHastatmakaKund (8 hand long) should be constructed.

Ancient scripture described Vedic mathematical formulation for construction and validation of constructed Yagya kund. This Vedic ritual Yagya included very stringent mathematical measurements for every part of the ritual. The size of Mandap is dependent on the size of the Kund and the size of the Kund is dependent on the number of offerings. The number of offerings is further dependent on the spiritual penance (Dharm-anushthan) it was meant for.

The shape of Kund depends on the purpose of Yagya but the size of the kund depends on the number of offerings to be sacrificed or burned in the sacred fire. In a recentstudy conducted by M. Jairam, the author reconfirmed the scriptural fact thatirrespective of shapes, all these Yagya kunds have same area (1). Hence, based on this information, the present research hypothesize that the volume should be equal for all different shaped kunds. In the present study, BhuHastatmakaKund (1 hand long) is considered for calculations to find the volume of different shape of the Kund of 1 hast (24 angul).

Methods

Ancient calculations used as a seed in the study

The classic book on Yagya kund construction used as a base in the present study is Kundark (4). The book describes the dimensions for constructions of different shapes in the unit – ‘angul’. The ancient method use circle as a starting point for constructing any shape of the kund. The book gave diameter for each shape of the kund for drawing first circle and then in the book, dimensions for sides were calculated based on vedic mathematics (Table 1, column 2 and column 3).

Lotus and circle shaped kunds have same inside volume structure. For all different shaped kunds of 1 hast (24 angul), the height of the kund is same, that is, 24 angul. Inside structure of all kunds have reverted pyramid shape, that is, the top surface is bigger and the lower surface is smaller. The base of the kund for 1 hast kund is always ¼ of the upper diameter or dimensions (6).

Modern calculations & formulas

Based on the diameter and dimensions of sides of different shape kunds given in the book kundark, the present study calculated volumes using mathematical formulas used in the modern time. All the calculations areperformed manually as well as using Microsoft excel. The formulations to calculate the volume for circular kund, semicircular kund, square and lotus kund were taken from aweb application (5) and for triangular, pentagonal, hexagonal, heptagonal and octagonal kund were taken from the study by Bullen, P.S (7).

The ancient units of measurement of length was 'angul' which was converted into 'cm' and calculations for kund volumes were performed. It is important to note that diameters and dimensions described in the scriptures weregiven in the ancient units, that is,'Angul.Yav.Yuka'. The unit in angul was then either directly applied in the mathematical formulas and the outcome in unit angul was converted into the cm or angul was converted into cm first and then applied into the required formulas.

Results

Figure 1 described inside structure of different shaped kunds. The sides are also shown in these figures with symbols or letters which are further described in the Table 1 along with formulas. The first column describes types of kund. The dimensions of circular kund and lotus kund are same therefore, they are represented by one figure only, rest all are calculated individually. The second column lists the diameter in 'Angul.Yav.Yuka' which is given in book kundark (4). The third column consists of dimensions of sides which are also directly taken from book kundark as seed units to do further calculations using mathematical formulas (4).

Calculations of volume using modern formulas

The volumes are calculated using two different measurements units. In the First one, the sides in angul (column 3) are applied into the formulas (column 8 & 9), and the volume is calculated in $(\text{angul})^3$ (column 4), which is further converted into the cm^3 by multiplying with $(1.9)^3$ (column5). Here, one angul is considered as 1.9 cm as given in J. Motlani (1) and angul is converted into cm by multiplying 1.9.

In the second one, sides in angul (column 3) are first converted into cm (column 6), and applied into the formulas and then the volume is measured in $(cm)^3$ (column 7). The value of volume calculated using both the approaches (column 5 and 7) are same, which showsthat calculations obtained same results and are accurate regardless of units applied.

Comparing volumes of different shaped kunds

The volume calculated is further compared in order to test the underlying hypothesisthat the volume of different shapes is same. Considering the circle Kund as a basis for comparison, the difference of volume among all kunds are calculated and described in the last column of Table 1.

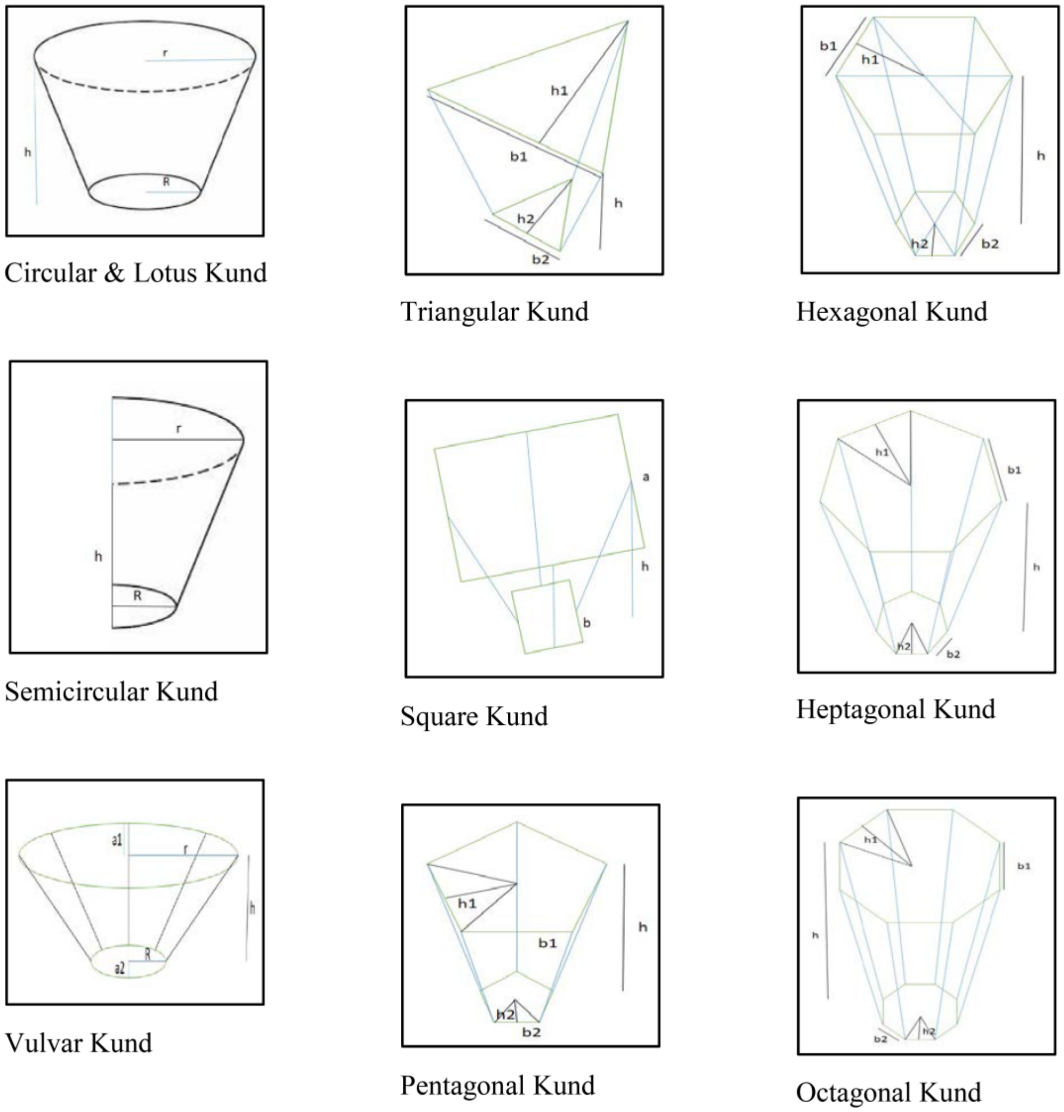

Figure 1.Inside structure of Kunds of different shapes aloing with sides and their symbols.

Type of kund	Diameter (Angul.Yav. Yuka) (4)	Sides (Angul) (4)	Volume (Angul^3)	Volume (cm^3) [Angul* (1.9^3)]	Sides in (cm) [Angul*1.9]	Volume (cm^3) [from sides in cm]	Formula used	Description for formula	Difference from circular kund (%)
Circular & Lotus	27.0.5	R-3.38 r-13.54	6045	41441.34	R-6.422 r-25.72	41441.34	(1/3*π*h)*(R^2+r R+r^2)	R-radius of inner circle r- radius of outer circle	0.00
Semi-circular	38.2.3	R-4.78 r-19.145	6043	41428.43	R-9.08 r-36.37	41428.43	([1/3*π*h)*(R2+r R+r2)]/2	R- radius of inner circle r- radius of outer circle	0.03
Vulvar	30.2.0	a1-15.05 b1-13.09 a2- 3.76	6497	44542.38	a1-28.59 a2-7.144 b1- 24.87	44542.38	((π*b1*h)/(3*a1)) *(a1^2+(a1*a2)+ a^2)	a1- upper elliptic perpendicular a2- lower elliptic perpendicular b1 half of base	7.48
Tringonal	42.1.0	b1-36.5 h1-31.5 b2-9.125 h2- 7.87	6035.6	41398.45	b1-69.35 h1-59.85 b2-17.33 h2- 14.953	41398.45	(h/3)*(b1a+√b1a* b2a+b2a)	h1-perpendicular of base 1 b1- side of base 1 b1a- area of base b1 h2- perpendicular of base 2 b2- side of base 2 b2a- area of base 2	0.10
Square	24.0.0	a-24 b-6	6048	41483.23	a-45.6 b-11.4	41483.23	(h/3)*(a2+ab+b2)	a-side of upper square b- side of base square	0.10
Pentagonal	31.1.0	b1-18.29 h1-12.75 b2-4.725 h2- 3.18	6149.53	42171.44	b1-34.75 h1-24.225 b2-9.968 h2- 6.04	42171.44	[(h/3)*(b1a+√b1a b2a+b2a)]5	h1- perpendicular of base 1 b1- side of base 1 b1a- area of base b1 h2- perpendicular of base 2 b2- side of base 2 b2a- area of base 2	1.76
Hexagonal	29.6.0	b1- 14.87 h1-12.87 b2- 3.71 h2- 3.21	6027	41322.93	b1- 28.25 h1-24.45 b2- 7.049 h2- 6.099	41322.93	[(h/3)*(b1a+√b1a b2a+ b2a)]6	h1- perpendicular of base 1 b1- side of base 1, b1a- area of base b1 h2- perpendicular of base 2 b2- side of base 2, b2a- area of base 2	0.29
Heptagonal	29.0.0	b1- 12.62 h1-13 b2- 3.15 h2- 3.25	6026.36	41344.95	b1- 23.98 h1-27.7 b2- 5.99 h2- 6.175	41344.95	[(h/3)*(b1a+√b1a b2a+b2a)]7	h1- perpendicular of base 1 b1- side of base 1, b1a- area of base b1 h2- perpendicular of base 2 b2- side of base 2, b2a- area of base 2	0.23
Octagonal	28.4.0	b1-10.95 h1-13.5 b2-2.73 h2- 3.40	6315	42613.38	b1- 20.81 h1-25.88 b2- 5.187 h2- 6.46	42613.38	[(h/3)*(b1a+√b1a b2a+b2a)]8	h1- perpendicular of base 1 b1- side of base 1, b1a- area of base b1 h2- perpendicular of base 2 b2- side of base 2, b2a- area of base 2	2.83

Table 1. Mathematical details of volume calculations for different shaped kunds i.e. volume in angul and cm, formula used, sides in angul and cm, and the difference in volume of various kund. *h is 24 angul and 45.6 cm and is the height of the kund, which is same for all shapes.

Volume of a circular kund and lotus kund

The circular kund looks like a half cone (See Figure 1), therefore the formula used would be the same used for half of the cone (frustum), which is $1/3*\pi*h(R^2+rR+r^2)$, Where π is 22/7; r istop surface radius of cone, R is radius of base come and h isthe depth of the kund.

As per the kundark shlok 3, the vyas (diameter) of the circular kund is 27.0.5 Angul. Yav.Yuka. This is actually the diameter of upper circle. Yav is 1/8th of angul and yuka is 1/64 of angul, therefore diameter 27.0.5 Angul.Yav.Yuka = 27+ 0/8+ 5/64 angul = 27+0+0.07 = 27.07 angul. Thus, the radius of upper circle R is 27.07/2= 13.535 angul.

The base of the kund for 1 hast kund is always ¼ of the upper diameter (6). Therefore, the base diameter would be 6.75 angul and radius r would be 6.75/2 angul = 3.37 angul. The depth (or height) of the 1 hast (24 angul) kund for any shape described in the scripture is 24 angul (6).

The formula used for circular kund is $1/3*\pi*h(R^2+rR+r^2)$. Putting these values into the formulation, we get $\{1/3*22/7*\ 24\ [(13.5)^2+13.5*3.37\ \ (3.37)^2]\}$ = 6045angul. 1 angul is equal to 1.9 cm, by multiplying to 6046.60 angul to the cube of 1.9 then volume would be 41441.34 $(cm)^3$. By converting the values of radius and height into cm (refer the table 1), the volume would be 41441.34$(cm)^3$. It confirms that the calculations are accurate (Table 1).

Volume of Chandra kund (semicircular kund)

The diameter of semicircular kund is 38.2.3 Angul.Yav.Yuka and R is 4.78, r is 19.145 and h is 24 (Figure. 2). Putting these values in formula $[1/3*\pi*(R^2+rR+r^2)]/2$, value obtained is 6043 angul, which is only 0.03% deviated from the circular kund, very negligible (Table 1).

Volume of vulvar kund (yoni kund)

Vulvar kund looked like a truncated elliptic cone (Figure. 1), the formula used for calculating its volume is $\pi b1h/3a1(a1^2+a1a2+a2^2)$. By substituting the values a1 as 15.05, b1 as 13.09, a2 as 3.76 angul into formulation, volume obtained is 6497 angul. This value is deviated from circular kund by 7.48% which is a significant deviation (Table 1). The main reason for this deviation is inappropriate approximation of volume formula. The formula is obtained by dividing the entire volume into components and each of the components is assumed to be symmetric. This would be clearer in the pentagonal shaped kund.

Volume of triangular kund

The triangular kund looks like triangular frustum (Figure. 1) but with the same height. Therefore, the formula used for calculating its volume is same as formula used to calculate the volume of frustum, that is $[h/3(b1a+\sqrt{}b1ab2a+\ b2a)]$7. The diameter for the triangular kund is 36.4.0 Angul.Yav.Yuka and b1 is 36.5, h1 is 31.5, b2 is 9.125, h2 is 7.87 and h is 24 angul. Where h1 is perpendicular to base 1, b1 is the side of base 1,b1a is the area of base b1, h2 is perpendicular to base 2, b2 is the side of base 2, b2a is the area of base 2 and h is the height of Yagya kund. The obtained volume of triangular kundis 6035.6 angul. It was showing slight deviation from circular kund which is 0.16% (Table 1).

Volume of square kund

Square kund looks like an inverted pyramid (Figure. 1). Therefore, the formula used for calculating the volume would be the same used to calculate the volume of pyramid which is $(1/3*(a^2+ab+b^2)*h)$. Here, 'a' is the side of upper square which is 24 angul and 'b' is the side of lower or base square which is 6 (¼ of upper square), and the height for 1 hast kund is 24 angul. By substituting the values in the formula, obtained volume of square kund is 6048 angul, which has 0.10% difference from the volume of circular kund (Table 1).

Volume of pentagonal kund

The structure of pentagon kund looks like pentagon frustum but with the same height (Figure. 1). In order to establish a formula for its volume, the pentagon kund is divided into 5 parts consisting of 5 equilateral triangles. Using the same formula given for triangular kund, the volume of 1 triangular frustum is calculated and it was multiplied by 5to obtained formula for pentagon shaped kunds, that is, [h/3(b1a+√b1ab2a+b2a)]5.

The diameter for pentagon kund is 18.2.3 Angul.Yav.Yuka unit and sides for the triangular frustum in pentagon kund are as b1=18.29, h1=12.75, b2=4.725, h2=3.18 and h=24 angul. By applying the values into the formula, we obtain 6149.53 angul volume of the pentagon kund. The volume is deviated by 1.67 % from circular kund (Table 1). Although, the reason of the deviation in all the cases is the same but it would be easy to understand from the design of pentagon kund. Note that while deriving the formula for volume, it was assumed that the pentagon is made up of 5 equilateral triangular frustums which is not an exact approximation. This is because these triangular frustums may not be equilateral. It can be easily seen by observing the angles of a pentagon.

Volume of hexagonal kund

The volume of hexagonal kund (Figure 1) is calculated using the same approach as used for pentagonal kund, the only difference is the increment of the volume of triangular frustum by 6 times. Therefore, the formula is [h/3(b1a+√b1ab2a+b2a)]6.

The diameter for hexagon kund is 14.7.0 Angul.Yav.Yuka and sides for the triangular frustum in hexagon kund are b1=14.87, h1=12.87, b2=3.71, h2=3.21, and h=24 angul. By applying the values into the formula, the obtained volume is 6027 angul which was deviated by 2.56 % from circular kund (Table 1).

Volume of heptagonal kund

The volume of a heptagonal kund (Figure 9) is calculated using the same approach as used for pentagonal kund, the only difference is the increment in the volume of triangular frustum by 7 times. Therefore, the formula used is [h/3(b1a+√b1ab2a+b2a)]7. The diameter for heptagonal kund is 12.5.0 Angul.Yav.Yuka and the sides for the triangular frustum in heptagonal kundareb1=12.62, h1=13, b2= 3.15, h2=3.25 and h=24 angul. By applying the values into the formula, the obtained volume will be 6026.36 angul (Table 1).

Volume of octagonal kund

The volume of octagonal kund (Figure 1) is calculated using the same approach used for pentagonal kund, the only difference is the increment to the volume of triangular frustum by 8 times. Therefore, the formula used was [h/3(b1a+√b1ab2a+b2a)]8. The diameter for octagonal kund is 10.7.5 Angul.Yav.Yuka and the sides for the triangular frustum in octagonal kund are b1=10.95, h1=13.5, b2=2.73, h2=3.40, h=24 angul. By applying the values into the formula, the obtained volume of kund is 6315angul. The deviation from circular kund is 2.83% (Table 1).

Discussions

The present study analyzes the volume of different shapes listed in the literature of Yagya. In order to calculate their volumes, modern mathematical formulation is used instead of Vedic mathematics. In the present study, the volume of 1 hast Yagya kund (24 angul) for 10 shapes is calculated using the dimensions given in the scripture (book Kundark (4)) using available mathematical formulas. The study concludes that the volume differences for semi-circular kund, vulvar kund, trigonal kund, square kund, pentagonal kund, hexagonal kund, heptagonal kund, and octagonal are 0.03%, 7.48%, 0.10%, 0.10%, 1.76%, 0.29%, 0.23%, and 2.83% respectivelywhen compared with the circle shape kund. The differences in the volume of different shapes are below 0.3% except for vulvar, pentagonal and octagonal shapes, which were 7.48%, 1.76% and 2.83% respectively.The variation in the simple shapes like semicircle, square are very less, <0.1% but as the complexity in structure increases, the difference in volume also increases. The formulas used are simple, rather than considering integration and differentiation of these complex structures, the approximations are made by using thedimensions of top and bottom bases.The use of vigorous mathematical formulas explains the higher percentage variation in vulvar, pentagonal and octagonal shapes. Further studyis needed to address the small variation using complex mathematics formulas.

It is remarkable that even though these kunds had different shapes but had almost same volume. In ancient times, the number of offerings in fire required a perfect calculation to have uninterrupted procedure. The present study hinted out the possibility of same volumes for 1 hast kund which is applicable to 2 hast, 4 hast kunds etc.

The observation of the present study shows that the Indian literatures are written on the basis of strong scientific studies. The constructions of various kunds with different sizes and shapes listed in Indian ancient literature have a very strong scientific base.

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